Math 107 Exam #4

Use interval notation to write the set of all possible values for the slope of a line through (3,4) if that line does not go into the second quadrant.

Find a quadratic function $f(x)$ such that $f\left(\frac{-8}{3}\right) = f\left(\frac{2}{3}\right)$, the minimum value of f is -10 and $f(0) = 15$.

Graph $f(x) = x^2 - 5x - 25$. Label the vertex and intercepts with exact values.

Find the area and the perimeter of the rectangle as functions of the diagonal if one side of the rectangle is one-half of the diagonal.

Find the smallest vertical distance between the graphs of $y = x^2$ and $y = x - 1$.

6. The price of each item when x items are sold is given by $P(x) = 250 - 2x$ dollars for $0 \le x \le 125$. Write a function $I(x)$ which represents the income from selling x items and find the domain and range of $I(x)$.

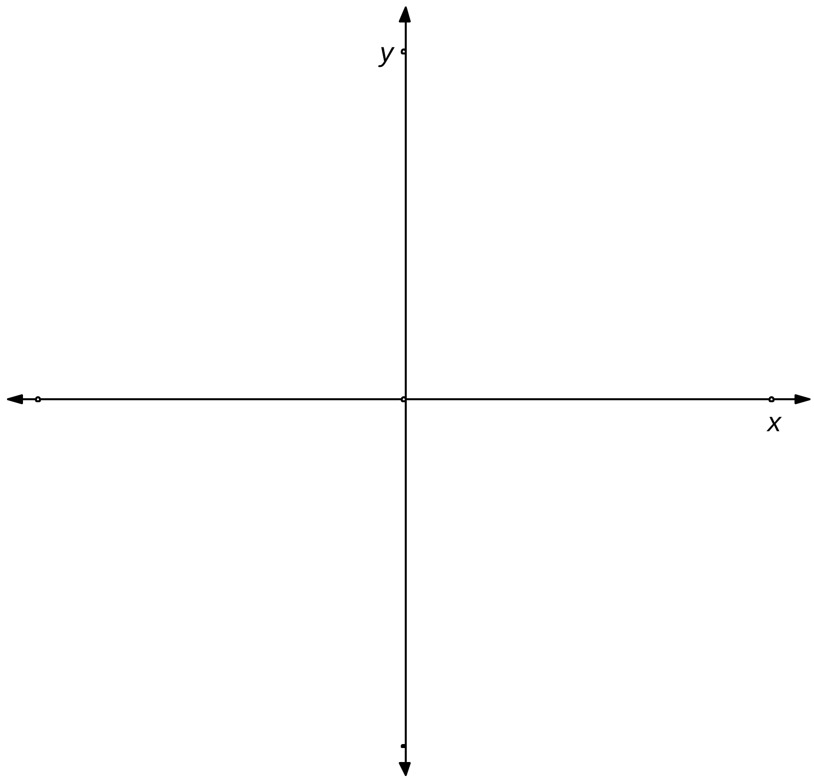
Sketch the graph $P(x) = x(x-20)^5(x+50)^8$ and label the intercepts.

Sketch the graph of $f(x) = \dfrac{x^3 - x}{50x(x-5)(x+5)}$. Find and label all intercepts and all asymptotes.

Create a rational function which passes through the points (5,0) and (-7,0), has vertical asymptotes of x = -2 and x = 3, a horizontal asymptote of y = 4 and a hole in the graph on the y-axis. Do not graph the function.

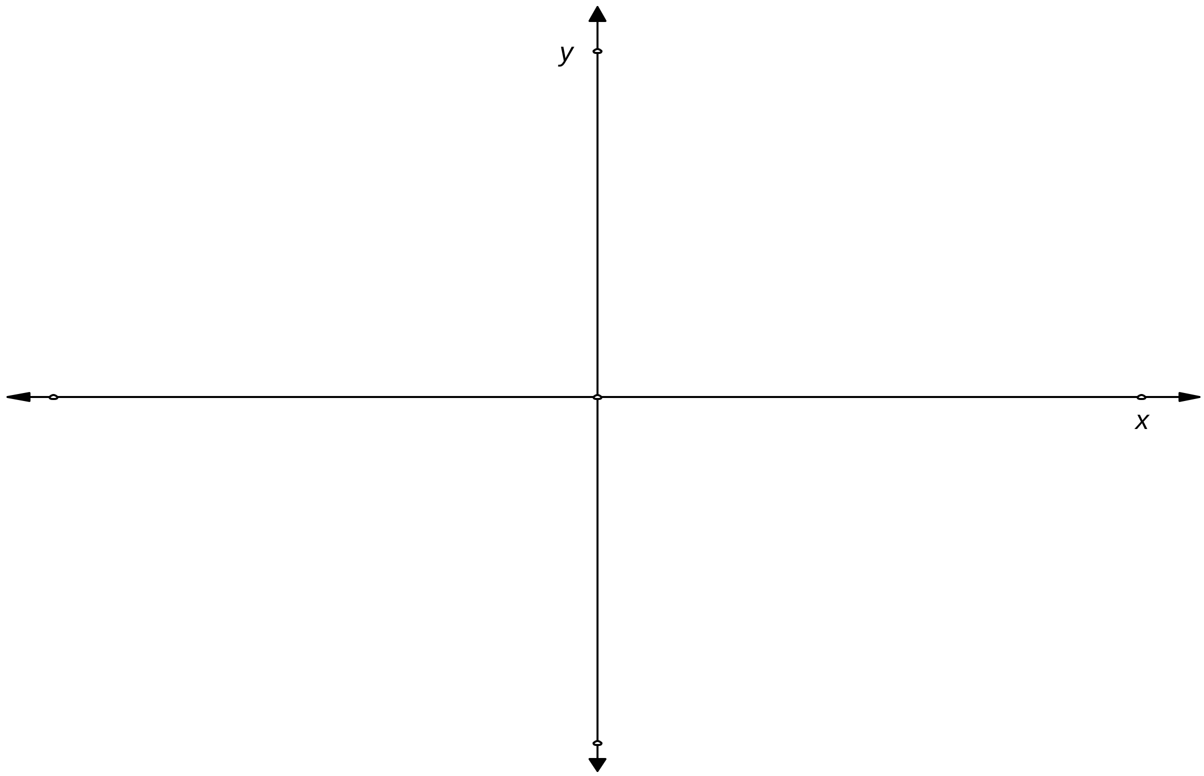
The graph of a quadratic function $f(x)$ passes through the the point $(-1, 1)$ and has a minimum value of $y = -4$ when $x = 3$. Find $f(x)$.

Sketch the graph of $y = 3 + 2x - x^2$ on the axes below. Specify the vertex, axis (line) of symmetry, maximum or minimum value of y, and the intercepts.



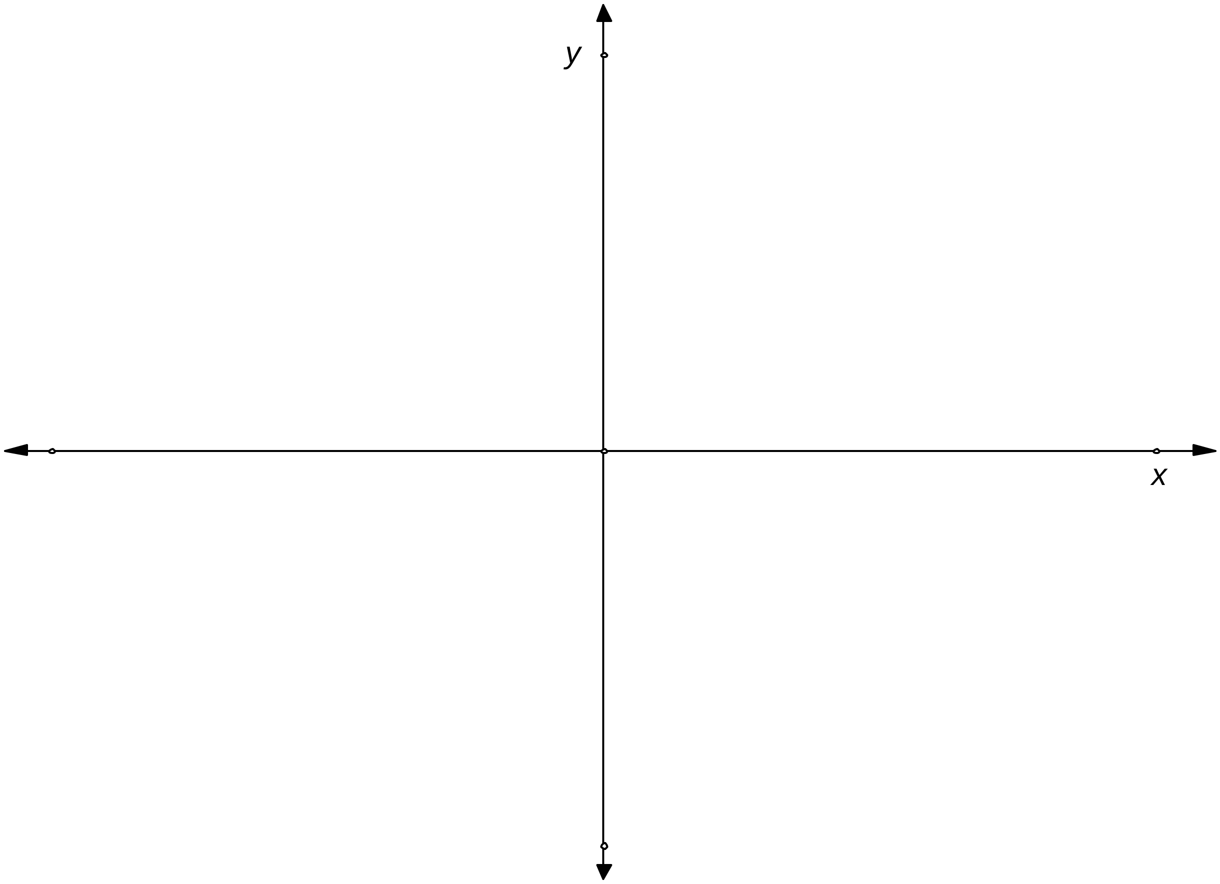
1.

Sketch the graph of the polynomial function $f(x) = x^2(x + \frac{15}{8})^2(x + 2)^3(25 - x)$ on the axes below. You do not need give an accurate scale, but specify all intercepts and give a good representation of the shape of the curve.



The revenue generated by selling x units of a certain product is given by $R(x) = -\frac{3}{5}x^2 + 150x$. Assume that R is in dollars. How many units of the product must be sold to generate this maximum (or minimum) revenue, and what is the maximum (or minimum) revenue possible?

Sketch the graph of $y = \dfrac{x^2 + x}{x^2 - \frac{1}{25}}$ on the axes provided. Specify the asymptotes (if any), point(s) where the graph cuts the horizontal asymptote (if applicable), and intercepts (if any).



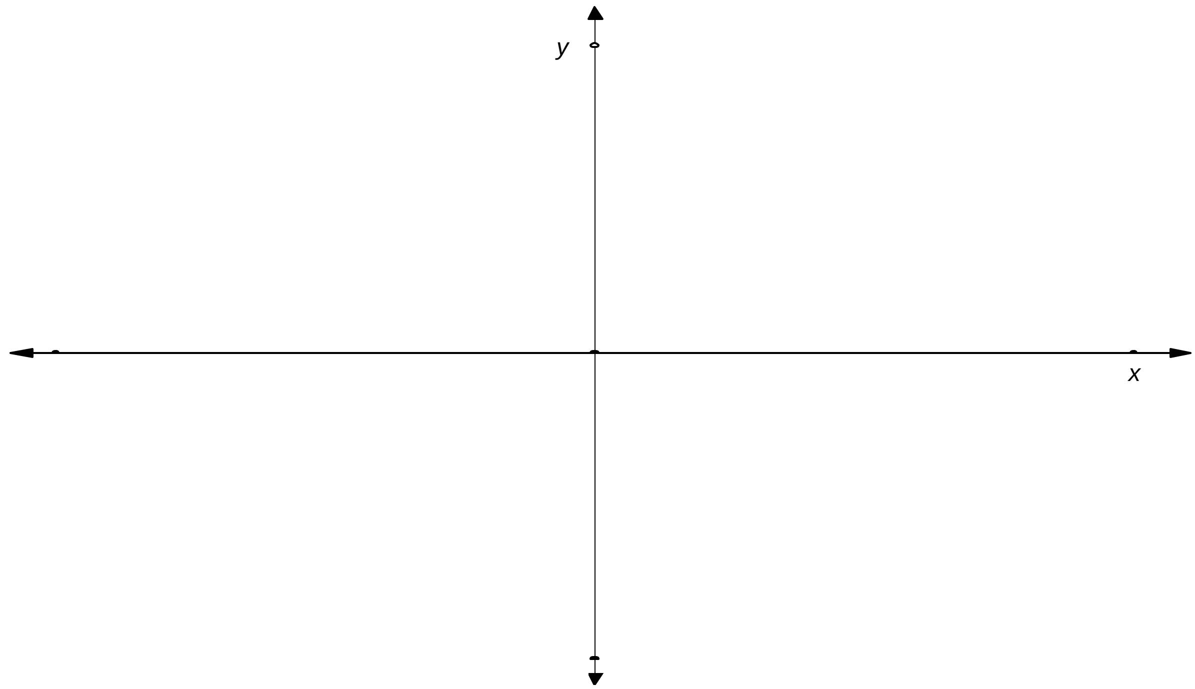
Find the maximum (or minimum) value of the function $f(x) = \sqrt{6 + 5x - x^2}$

Find two numbers adding to $\sqrt{2}$ such that their product is as large as possible.

Find an equation for a rational function that satisfies the following conditions:

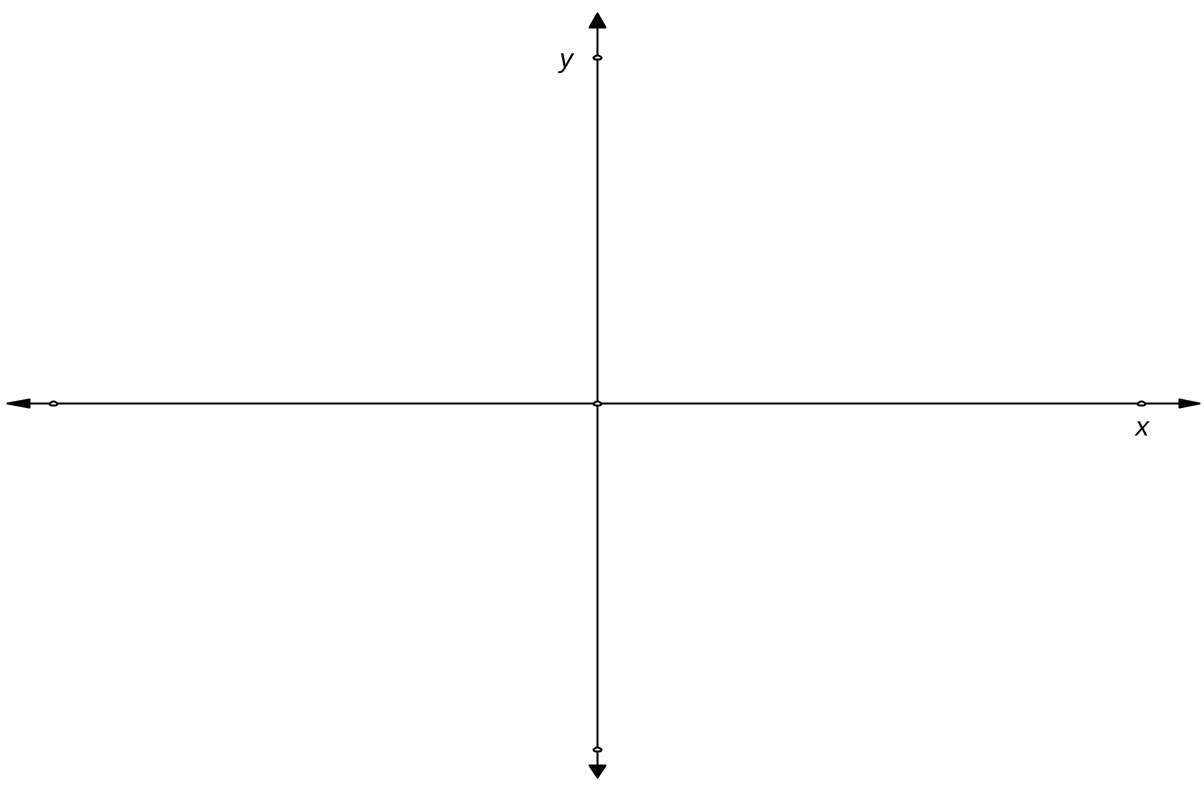
-
  - The graph has x-intercepts at $(2, 0)$ and $(-1, 0)$.
  - The graph has vertical asymptotes at $x = 1$ and $x = -2$.
  - The graph has a horizontal asymptote at $y = \frac{2}{3}$.
  - The graph has a hole at $x = 5$.

Sketch the graph of $y = \dfrac{x^2 + x - 6}{x - 3}$ on the axes provided. Specify the vertical and horizontal (or slant) asymptotes (if any); point(s) where the graph cuts the horizontal (or slant) asymptote (if applicable); intercepts (if any); and holes in the graph (if any).



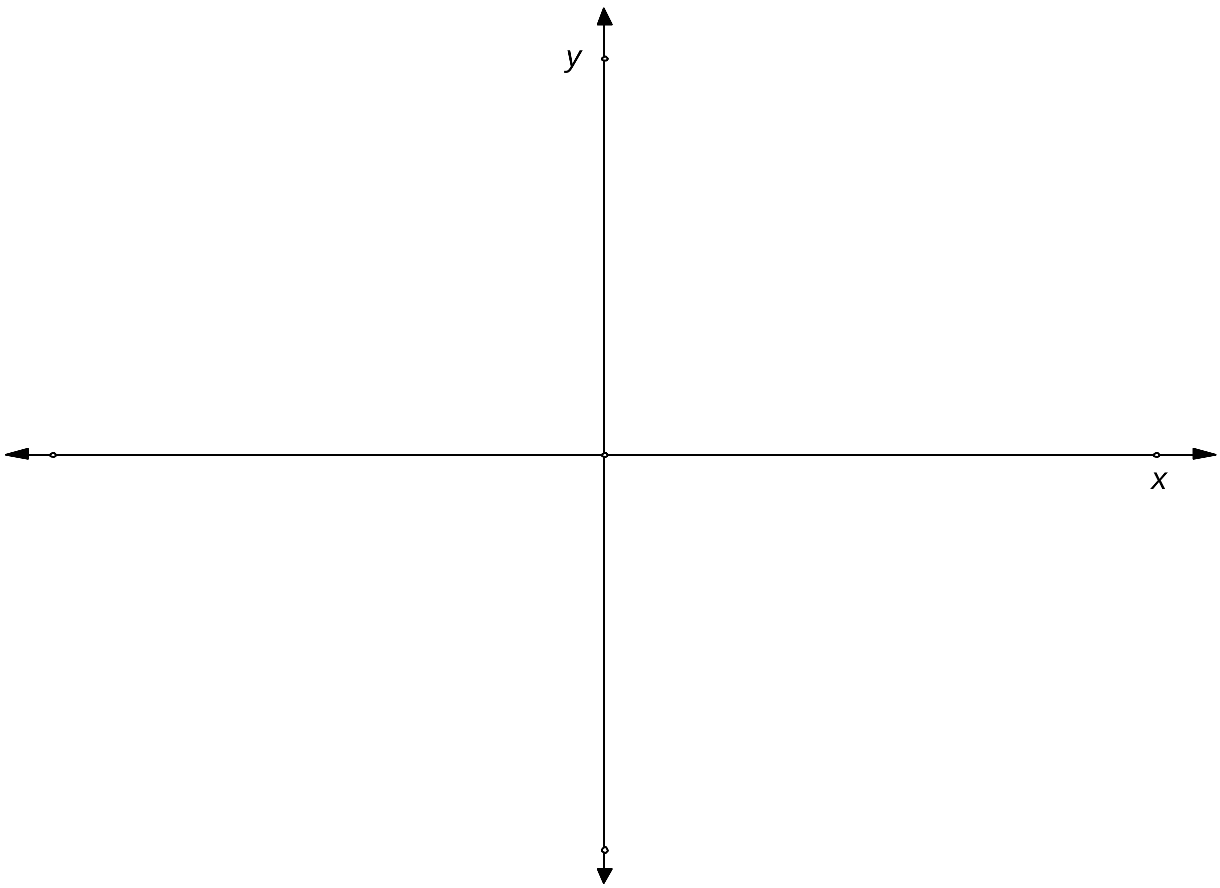
Find the quadratic function whose graph passes through the the point $(-1, 1)$, and whose vertex is at the point $(3, -4)$

Sketch the graph of the polynomial function $f(x) = -x^3(x + \frac{15}{8})^2(x + 2)^3(x - 25)$ on the axes below. You do not need give an accurate scale, but specify all intercepts and give a good representation of the shape of the curve.



The revenue generated by selling x units of a certain product is given by $R(x) = -\frac{3}{5}x^2 + 150x$. Assume that R is in dollars. What is the maximum (or minimum) revenue possible in this situation? How many units of the product must be sold to generate this maximum (or minimum) revenue?

Sketch the graph of $y = \dfrac{-x^2 - x}{x^2 - \frac{1}{25}}$ on the axes provided. Specify the asymptotes (if any), point(s) where the graph cuts the horizontal asymptote (if applicable), and intercepts (if any).



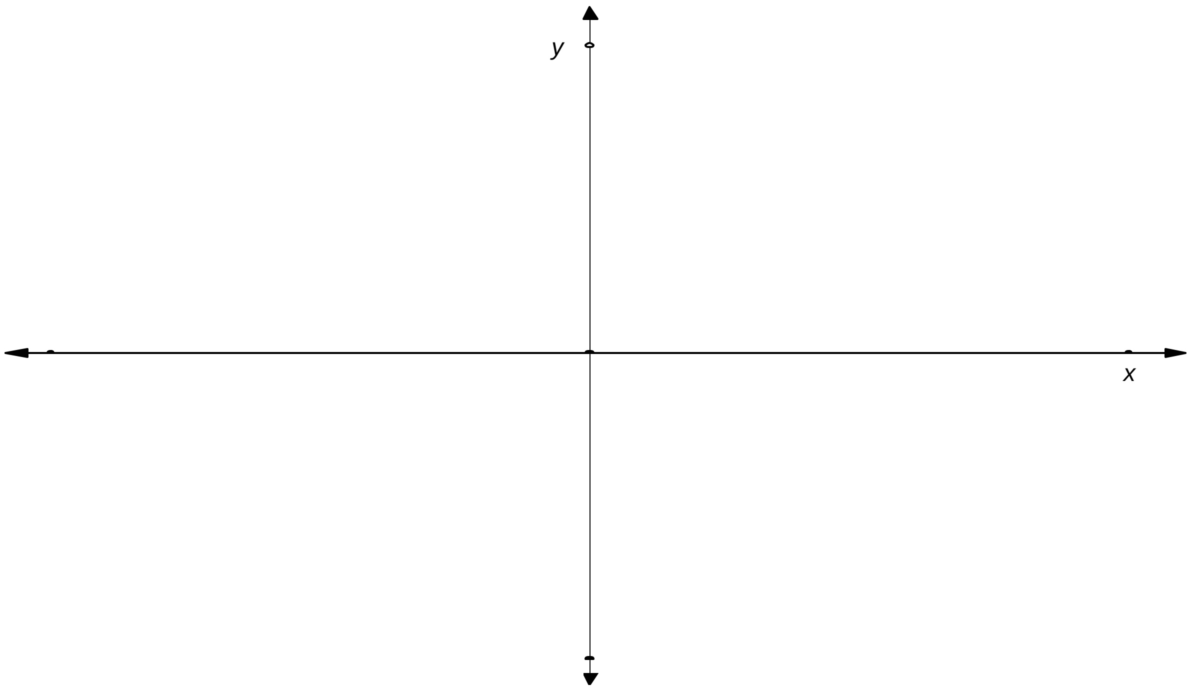
Find the coordinates of the point on the line $y = x + 2$ that is closest to the point $(-1, 2)$.

Find two numbers adding to $\sqrt{2}$ such that their product is as large as possible.

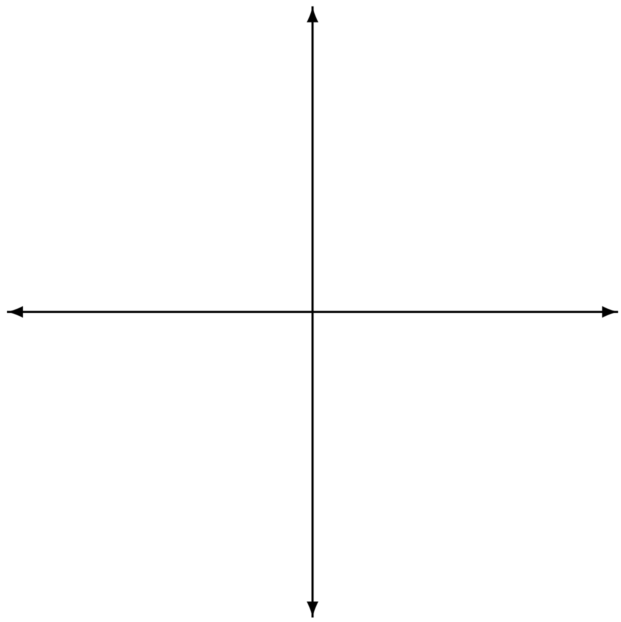
Find an equation for a rational function that satisfies the following conditions

2.
- The graph has x-intercepts at $(2, 0)$ and $(-1, 0)$.
- The graph has vertical asymptotes at $x = 1$ and $x = -2$.
- The graph has a horizontal asymptote at $y = \frac{2}{3}$.
- The graph has a hole at $x = 5$.

Sketch the graph of $y = \dfrac{x^2 + x - 6}{x - 3}$ on the axes provided. Specify the vertical and horizontal (or slant) asymptotes (if any); point(s) where the graph cuts the horizontal (or slant) asymptote (if applicable); intercepts (if any); and holes in the graph (if any).

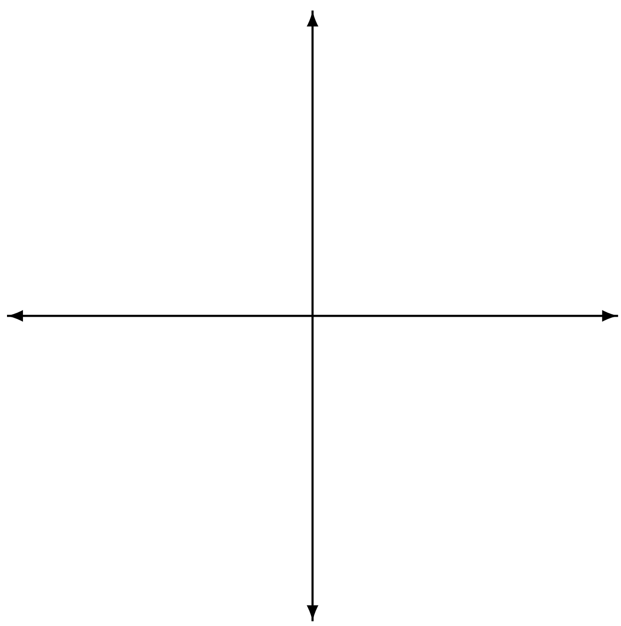

y
x

Let $f(x) = -x^2 + 7x - 2$. Find the x-intercepts, y-intercept, and vertex. (give exact answers, for example $\sqrt{2}$ rather than 1.414). Then graph the function.



Find a quadratic function $f(x)$ with x-intercepts $(2, 0)$ and $(-3, 0)$ such that $f(0) = 18$.

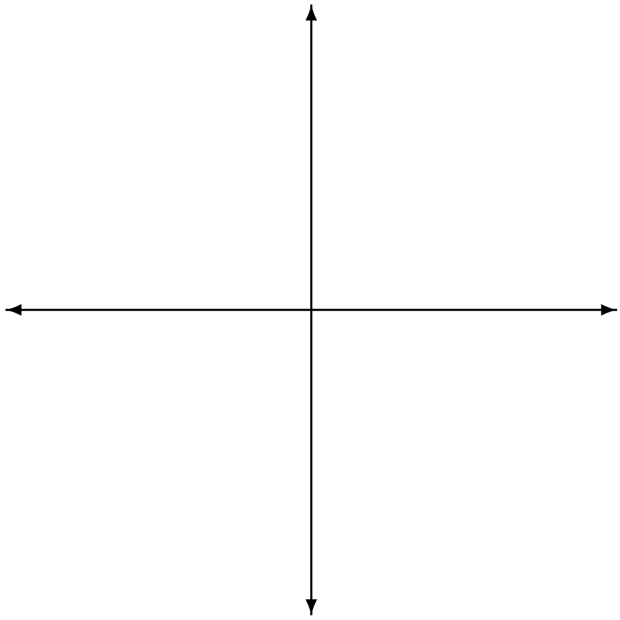
Let $g(x) = (2 - x)(x + 3)^3(x + 20)^2$. Find the x-intercept(s) and y-intercept. Then graph the function.



Let $f(x) = x^2 - 6x + 11$ and $g(x) = x^2$. Find the minimum of $g \circ f$.

A rancher wants to enclose a rectangular region using \$600 yards of fencing. However, one of the sides of the rectangle will not need fencing because it is next to a river. The sides perpendicular to the river will cost \$9 per yard and the side opposite the river will cost \$6 per yard. Determine the dimensions of the rectangle that will yield the largest rectangle.

Let $f(x) = \dfrac{x^2 - 4}{x + 1}$. Find the x-intercept(s), y-intercept, asymptotes, and graph the function.

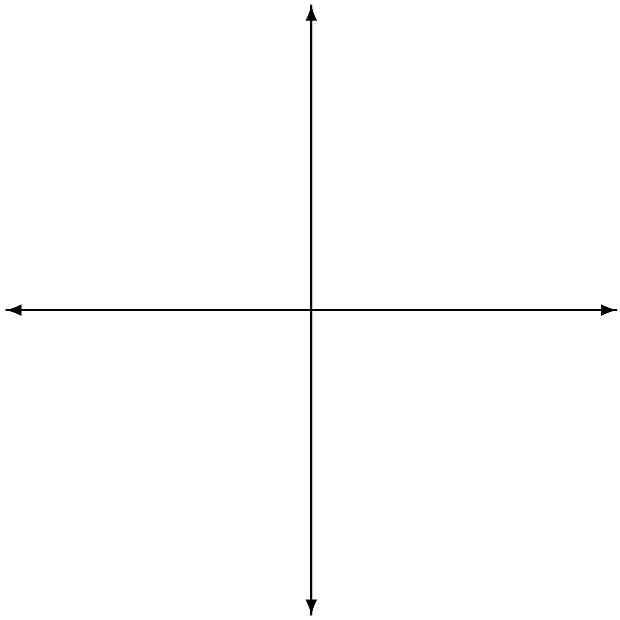


Construct a rational function with the following properties:

- x-intercepts: $(-2, 0)$ and $(3, 0)$
- vertical asymptotes: $x = -3$ and $x = 5$
- horizontal asymptote: $y = -2$
- a hole at $x = -5$

Find the range of $h(x) = -2x^2 + 6x$.

Let $g(x) = \dfrac{(x-2)^2}{x^2 - 99x - 100}$. Find the x-intercept(s), y-intercept, asymptotes, where the graph crosses the horizontal asymptote (if any) and graph the function. Do not draw the x-axis and y-axis with the same scale.



Find a quadratic function $f(x)$with vertex $(-3, 1)$ such that $f(-5) = 9$.

Let $f(x) = -2x^2 + 6x - 2$. Find the x-intercept, y-intercept, and vertex. (give exact answers, for example $\sqrt{2}$ rather than 1.414). Then graph the function.

Let $g(x) = (x+3)^2(1-x)^3(x-20)$. Find the x-intercept(s), the y-intercept, and the graph of $g(x)$.

Find the point on the curve $y = \sqrt{2x - 3}$ that is closest to $(5, 0)$.

The hypotenuse of a right triangle is 20 cm. Let x be one of the other sides of the triangle. Express the area of the triangle as a function of x.

Find all asymptotes (vertical, horizontal and slant) of the following function:

$$g(x) = \frac{2x^3 - 11x + 5}{x^2 - 3x - 18}$$

Construct a rational function with the following properties:

- x-intercepts: $(-2, 0)$ and $(3, 0)$
- vertical asymptotes: $x = -3$ and $x = 5$
- horizontal asymptote: $y = -2$
- a hole at $x = -5$

Find c so that the range of $f(x) = x^2 + 2x + c$ is $[\sqrt{2}, \infty)$.

Let $g(x) = \dfrac{50x^2}{x^2 - 2x - 3}$. Find the x-intercept(s), y-intercept, asymptotes, where the graph crosses the horizontal asymptote (if any) and graph the function. Do not draw the x-axis and y-axis with the same scale.