REVIEW FOR 054 PLACEMENT TEST

This Review is intended to give you an idea of what you will need to know to place into Math 055, Beginning Algebra. ***It should be a reminder of concepts and techniques you have learned in the past, rather than an introduction to new information.***

Use the practice questions at the end of the review to improve your skills. There is a key for the test. It will be most useful to you to complete the test before checking your answers. The review questions are more challenging than the Placement Test. Refer to the Review to help with questions you might have missed. The Learning Center also has textbooks you are welcome to check out if you want further examples.

PROPERTIES OF ADDITION

Identity Property of Addition

The sum of a number and zero is the original number. $a + 0 = a$

Commutative Property of Addition

Changing the order in which two numbers are added does not affect the sum.

$a + b = b + a$

Associative Property of Addition

When adding numbers, regrouping gives the same sum.

$(a + b) + c = a + (b + c)$

PROPERTIES OF MULTIPLICATION

The Identity Property of Multiplication

The product of any number and 1 is that number. $1a = a$

Multiplication Property of Zero

The product of any number and 0 is zero. $a0 = 0$

Commutative Property of Multiplication

Changing the order in which we multiply two numbers gives the same product.

$ab = ba$

The Distributive Property

Multiplying a factor by the sum of two number gives the same result as multiplying the factor by each of the two numbers, then adding the product.

$a (b + c) = ab + ac$

MULTIPLICATION/ DIVISION:

$3 \cdot 5 = 5 \cdot 3 = 15$

$5 \times 3 = 3 \times 5 = 15$ These are all different ways to represent 5 multiplied by 3.

$(5)(3) = (3)(5) = 15$

$\frac{12}{4}$, $12 \div 4$, $4\overset{3}{\overline{)12}}$ Are all different ways to represent 12 divided by 4.

ABSOLUTE VALUE:

The Absolute Value of a number is its distance from zero on the number line. It is represented by the symbol $| \; |$. $|-2| = 2$ Because negative two is two units away from zero on the number line.

WORKING WITH SIGNED NUMBERS

Negative Numbers:

A positive number is greater than zero. For example: $4 > 0$

A negative number is less than zero. For example: $-2 < 0$

Adding signed numbers:

If the numbers have the same sign, add the absolute values of the numbers and leave the sign.

$(-1)+(-2)=(-3)$

$(1)+(-2)=(-1)$

Multiplication and Division of Signed Numbers:

1. Multiply their absolute value.
2. If the numbers have the same sign, their product is positive; if the numbers have different signs, their product is negative.

$6\cdot6=36 \qquad (+6)(+6)=36$

$(-6)(-6);\ |-6|\cdot|-6|=\ 6\cdot6=36$ Since the signs are the same the result is positive

$(-5)(4);\ |-5|\cdot|4|=\ 5\cdot4=20$ Since the signs are not the same the result is negative

1. Division follows the same rules.

$3=\frac{9}{3};\quad \frac{|9|}{|3|}=\frac{9}{3}$

$2=\frac{-4}{-2};\quad \frac{|-4|}{|-2|}=\frac{4}{2}$ Since the signs are the same the quotient is positive 2.

$-5=\frac{-10}{2};\quad \frac{|-10|}{|2|}=\frac{10}{2}=5$ Since the signs are different, the quotient is negative 5.

EXPONENTS

An exponent (or power) is a number that indicates how many times another number (the base) is multiplied.

For example:

$3^2=3\cdot3=9$

$5^3=5\cdot5\cdot5=125$

$4^5=4\cdot4\cdot4\cdot4\cdot4=1024$

$(-1)^4=(-1)(-1)(-1)(-1)=1$

$(-2)^3=(-2)(-2)(-2)=-8$

$-3^3=-(3)(3)(3)=-27$ Notice here how the exponent doesn't apply to the negative sign.

ORDER OF OPERATIONS FOR SIMPLIFYING EXPRESSIONS:

1. Perform the operations within any grouping symbols ( ) or [ ].
2. Apply any exponents:
3. Multiplication/ Division: left to right
4. Addition/ subtraction: left to right

PERCENTAGES

Percent (%) means divided by a hundred.

To change a percent to a decimal : 75% = $\frac{75}{100}$ or $\begin{array}{r} .75 \\ 100\overline{)75} \end{array} = .75$

Hint: move the decimal two places to the left and drop the % sign.

To change a decimal to a percent: $0.75 \cdot 100\% = 75\%$

Hint: move the decimal two places to the right and add the % sign.

FACTORING

In a multiplication problem, the numbers that are being multiplied are called factors.

Prime Factors

A prime factor is any whole number greater than 1 that has only 1 and itself as factors. For example, 10=2x5 and 21=3x7. To find the prime factorization of a number, divide the number by a series of primes until the final quotient is a prime number.

Find the prime factorization of 80.

$80 \div 2 = 40$

$40 \div 2 = 20$

$20 \div 2 = 10$

$10 \div 2 = 5$

The prime factorization of 80 is 2x2x2x2x5

Greatest Common Factor (GCF)

The GCF of a group of numbers is the largest number that will divide each of the given numbers exactly.

Finding the GCF:

1) Write the prime factorization of each number in the group
2) Locate the prime factors that are common
3) The GCF will be the product of all of the common prime factors

Find the GCF of 20 and 30:

20 = 2x2x5 and 30 = 2x3x5 so the GCF of 20 and 30 is 2x5 = 10

Least Common Multiple (LCM)

1) Write the prime factorization of each number in the group.
2) Find all the prime factors that appear in any one of the prime factorizations.
3) Form the product of those prime factors using each factor the greatest number of times it occurs in any one factorization.

Find the LCM of 15, 21 and 60:

15 = 3x5, 21 = 3x7 and 60 = 2x2x3x5

The LCM is 2x2x3x5x7 or 420

FRACTIONS

Changing a mixed number to an improper fraction

1. Multiply the denominator by the whole number. $4\frac{5}{6}$; 6*4=24
2. Add the numerator to that product. $\frac{24+5}{}$
3. Write the sum over the original denominator to form the improper fraction $\frac{29}{6}$

Changing an improper fraction to a mixed number

1) Divide the numerator by the denominator
2) If there is a remainder, write it over the original denominator

$\frac{38}{7}$ $38 \div 7$ =5 with a remainder of 3 so $\frac{38}{7}=5\frac{3}{7}$

Adding and Subtracting fractions

If the denominators are the same, add the numerators and use the common denominator.

$$\frac{1}{6}+\frac{4}{6}=\frac{5}{6}$$

If the denominators are not the same:

1) Find the lowest common denominator (LCD)
   a) use the least common multiple (the smallest number that all the denominators will divide into) to determine what each denominator needs.
   b) Rewrite the fractions as equivalent fractions using the information found in 1.

$$\frac{2}{3}+\frac{1}{2}-\frac{3}{4}= \qquad ;\text{LCD for 3, 2, 4} = 12$$

$$\frac{2}{3}\times\frac{4}{4}+\frac{1}{2}\times\frac{6}{6}-\frac{3}{4}\times\frac{3}{3}=$$

$$\frac{8}{12}+\frac{6}{12}-\frac{9}{12}$$

2) Add the numerators, use the new denominator and simplify if necessary

$$\frac{8+6-9}{12}=\frac{5}{12}$$

When one of the numbers doesn't look like a fraction, make its denominator 1.

$$6 + \frac{2}{3} = \frac{6}{1} + \frac{2}{3} = \frac{3}{3}\times\frac{6}{1} + \frac{2}{3} = \frac{18}{3} + \frac{2}{3} = \frac{20}{3} = 6\ \frac{2}{3}$$

Multiplying fractions

1) Multiply numerators
2) Multiply denominators
3) Simplify

$$\frac{1}{2}\times\frac{3}{5}=\frac{1\times 3}{2\times 5}=\frac{3}{10} \qquad \frac{3}{2}\times\frac{4}{12}=\frac{3\times 4}{2\times 12}=\frac{12}{24}=\frac{1\times 12}{2\times 12}=\frac{1}{2}$$

Division of fractions

1) Write first fraction
2) Invert second fraction and change ÷ to *.
3) Multiply as above.

$$\frac{2}{3}\div\frac{3}{5}=\frac{2}{3}\times\frac{5}{3}=\frac{10}{9}=1\frac{1}{9}$$

ALGEBRA

A variable is a letter that represents an unknown number. A constant is a known number. An algebraic expression contains one or more variables and may contain any number of constants. An algebraic equation is a mathematical statement that two expressions are equal.

Simplifying Algebraic Equations

Adding and Subtracting Algebraic Expressions

1) Identify like terms (terms with the same variable and exponents)
   -2x + 4y + 5x – 3y
2) Addition is commutative.
   -2x + 5x + 4y - 3y
3) Distributive Property in reverse
   (-2 + 5)x + (4 - 3)y
4) Add like terms
   3x + y

Multiplying and Dividing Algebraic Equations

Multiplying:

Multiply the coefficients and the variables and then simplify

$$6x \cdot 2y = 12xy$$

Dividing:

Invert the divisor and multiply straight across and then simplify

$$9x \div 3y = \frac{9x}{1} \cdot \frac{1}{3y} = \frac{9x}{3y} = \frac{3x}{y}$$

Exponents in Algebra

$\underbrace{x \cdot x \cdot x \cdot x \cdots x}_{n\ times} = x^n$ for example $x \cdot x \cdot x = x^3$

$a^m \cdot a^n = a^{m+n}$ $\qquad x^5 \cdot x^1 = x \cdot x \cdot x \cdot x \cdot x \cdot x = x^6$

or $\quad x^5 \cdot x = x^{5+1} = x^6$

$\frac{a^n}{a^m} = a^{n-m}$ $\qquad \frac{r^5}{r^2} = \frac{r \cdot r \cdot r \cdot r \cdot r}{r \cdot r} = r \cdot r \cdot r = r^3$

$\qquad \frac{r}{r^2} = r^{5-2} = r^3$

$\left(a^n\right)^m = a^{nm}$ $\qquad \left(s^2\right)^3 = s^2 \cdot s^2 \cdot s^2 = s^{2+2+2} = s^6$

$a^{-n} = \frac{1}{a^n}$ $\qquad a^{-2} = a^{0-2} = \frac{a^0}{a^2} = \frac{1}{a^2}$ $\qquad$ remember $a^0 = 1$

WORD PROBLEMS

Phrases to watch for:

Addition	**Subtraction**	**Multiplication**	**Division**
x **plus** four	n **minus** 3	¾ **times** y	z **divided** by 5
x **increased by** 4	n **decreased** by 3	the **product** of ¾ and y	the **quotient** of z and 5
the **sum** of x and 4	the **difference between** n and 3	¾ **of** y	z **over** 5
4 **more than** x	3 **less than** n		
$x + 4$	$n - 3$	¾y or $\frac{3y}{4}$	$z \div 5$ or $\frac{z}{5}$

Ideas for solving word problems:

1. Identify the variable. Look for phrases like: what is, how many, how much, how far, how long. Let x (or variable of your choice) represent the question or number you will be trying to find.
2. Identify the information given and see how it relates to the variable or a known formula.
3. Determine what formulas or concepts you might need. Draw pictures. If the question asks for dimensions of a shape, draw the shape. Draw diagrams of the situations. Write out the equations you might use to solve the problem.
4. Try breaking up the question into a chain of smaller questions.
5. Try organizing the information in a table.

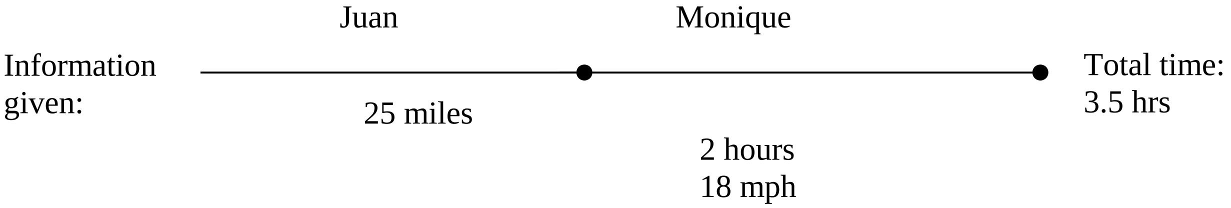
Example Word Problems

1. What number increased by four is eleven?
    a. "what number" is represented by x
    b. "increased by four" $x + 4$
    c. "is eleven" $x + 4 = 11$
    d. solve for x

2. Three-fourths of a number decreased by five is seven. Find that number.
    a. "a number" is represented by x
    b. "three-fourths of a number" $\frac{3}{4}x$
    c. "decreased by five" $\frac{3}{4}x - 5$
    d. "is seven" $\frac{3}{4}x - 5 = 7$
    e. solve for x

3. The area of a rectangular pen is to be 450 ft. If the length of the pen is 15 ft, what should the width be?
    a. "area of a rectangular pen" area = length times width = $l \cdot w$
    b. "is to be 450 ft" $l \cdot w = 450$ ft
    c. "if the length… is 15 ft" $l = 15$ft so, $15 \cdot w = 450$ ft
    d. "what should the width be?" $15w = 450$ ft
    e. solve for w

4. In a bicycle relay, Juan bikes twenty-five miles, then Monique bikes for two hours at eighteen mph. If Juan and Monique's total time is three-and-a-half hours, what is Juan's speed?



If the total time is 3.5 hours and Monique biked for 2 hours, Juan biked for 1.5 hours. Juan biked 25 miles for 1.5 hours. $Distance = rate \cdot time$ therefore $25 miles = 1.5 hours \cdot rate$. Solve for the rate.

$25 = 1.5r$ $\quad\quad$ $\frac{25}{1.5} = r = 16.67 mph$

054 PLACEMENT TEST PRACTICE QUESTIONS

These questions are intended to help you practice and review. They are more challenging than the questions you will see on the Placement Test Exam.

1.) Simplify: $4^2 + 5(10 + 15 \div 5) - 4$

2.) Write an equivalent expression for: $\frac{4}{5} = \frac{?}{25}$

3.) 23% of what number is 161?

4.) Divide $\frac{38x^4y^7 - 57x^3y^5}{-19x^2y^6}$

5.) Solve for x: $-3x - 3(2x - 7) = 39$

6.) Convert $\frac{7}{40}$ into a percent.

7.) Simplify: $(-3)^3$

8.) Find a number such that the sum of that number and 8, divided by 3 is equal to 5.

9.) Solve for x: $\frac{x}{4} - \frac{x}{5} = \frac{1}{2}$

10.) Simplify: $[17 + (34 + 95 \div 5)] \div 35 - 30$

11.) $12\frac{1}{3} \div 8\frac{2}{9}$

12.) Simplify: -3^4

13.) Before downsizing, Company A had 625 employees. After downsizing this company had 525 employees. What was the percent decrease in the number of employees?

14.) Simplify: $-\frac{3}{5} \div \frac{3}{-4} \div \frac{-6}{5}$

15.) Simplify: $\frac{(-x)^5}{(-x)^8}$

16.) If a recipe calls for $3\frac{1}{2}$ cups of flour and 2 eggs. How much flour would you need if you use 7 eggs?

17.) Simplify: $3yz(y + z) - 2yz(z - y)$

18.) Solve for x: $5(2x + 3) - 3(x - 3) = 12$

KEY FOR 054 PRACTICE QUESTIONS

1. 77
2. 20
3. 700
4. $\frac{-x(2xy^2-3)}{y}$ *or* $-2x^2y-\frac{3x}{y}$
5. x = -2
6. 17.5%
7. –27
8. x = 7
9. x = 10
10. –28
11. $\frac{3}{2}$
12. –81
13. There is a 16% decrease in the number of employees.
14. $-\frac{3}{2}$
15. $-\frac{1}{x^3}$
16. $12\frac{1}{4}$ cups of flour
17. $5y^2z-yz^2$
18. $x=-\frac{12}{7}$