

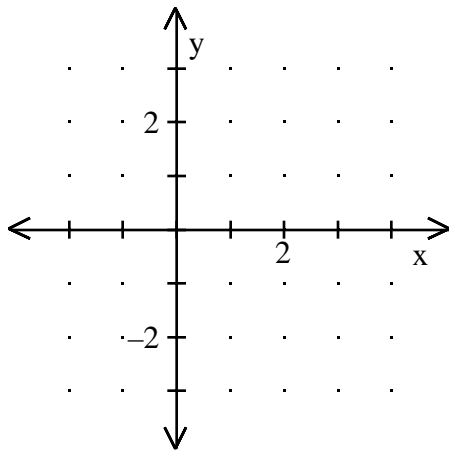
Math 107 Exam #3

Consider the functions  $f = \{(-2,3),(-1,0),(0,5),(1,2)\}$  and  $g = \{(0,-2),(1,0),(2,3),(3,1)\}$ . Write the sets of ordered pairs that represent the following functions.

(a)  $\frac{f}{g}$

(b)  $f \circ g$

Graph the function  $f(x) = \begin{cases} |x| & \text{if } -2 \leq x < 1 \\ -\sqrt{x} & \text{if } 1 < x \leq 4 \end{cases}$  and state the domain and range in interval notation. Is  $f(x)$  a one-to-one function? Explain your answer.

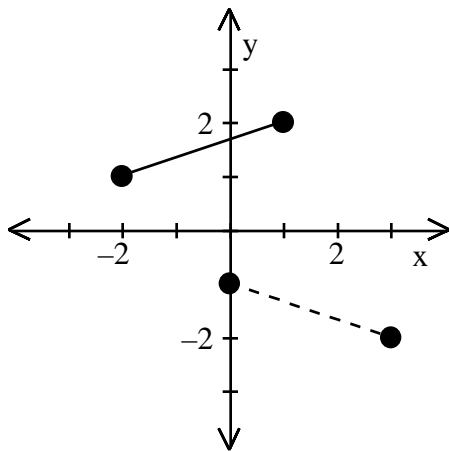


Find the average rate of change  $\frac{f}{x} = \frac{f(1+h) - f(1)}{h}$  of the function  $f(x) = x - \frac{1}{x}$  on the interval  $[1, 1+h]$ . Simplify as much as possible.

4. For the functions  $f(x) = 2x - 1$  and  $g(x) = 3x + B$ , find the value of  $B$  so that  $f(g(x)) = g(f(x))$ .

For  $f(x) = x^2 + 1$  and  $g(x) = \sqrt{x-1}$  find and simplify  $(f \circ g)(x)$  and  $(g \circ f)(x)$  and state their domains in interval notation.

If the solid line segment represents  $y = f(x)$ , describe in words the transformations that change the solid line segment into the dotted line segment and write an equation in the form of  $y = \pm f(\pm x + c) + d$  that describes the dotted line segment as a transformation of  $y = f(x)$ .



If  $f(x) = \frac{2x}{x-3} + 1$ , find and simplify  $f^{-1}(x)$  and write the domain and range of  $f(x)$  and  $f^{-1}(x)$  in interval notation. Also find and simplify  $(f \circ f)(3x^2 - 17x + 59)$ .

If  $f^{-1}(0) = 10$ , find the value of  $t$  that satisfies the equation  $f \frac{5-2t}{3t+1} = 0$ .

If  $(1, 7)$  is a point on the graph of a one-to-one function  $y = f(x)$ , then find a point in the first quadrant on the graph of  $y = -3f^{-1}(2x^2 - 1) + 9$ .

The electrostatic force,  $F$ , of repulsion between two positive electrical charges is jointly proportional to the two charges,  $q_1$  and  $q_2$ , and inversely proportional to the square of the distance,  $d$ , between the two charges.

(a) Express  $F$  in terms of  $q_1$ ,  $q_2$  and  $d^2$ .

(b) If both positive charges are tripled and the distance between them is tripled, what effect does this have on the electrostatic force of repulsion?

Figure 1 below is the graph of a function  $g(x)$ , the points P, Q, R and S lie on the this graph. Refer to this figure for the next four questions

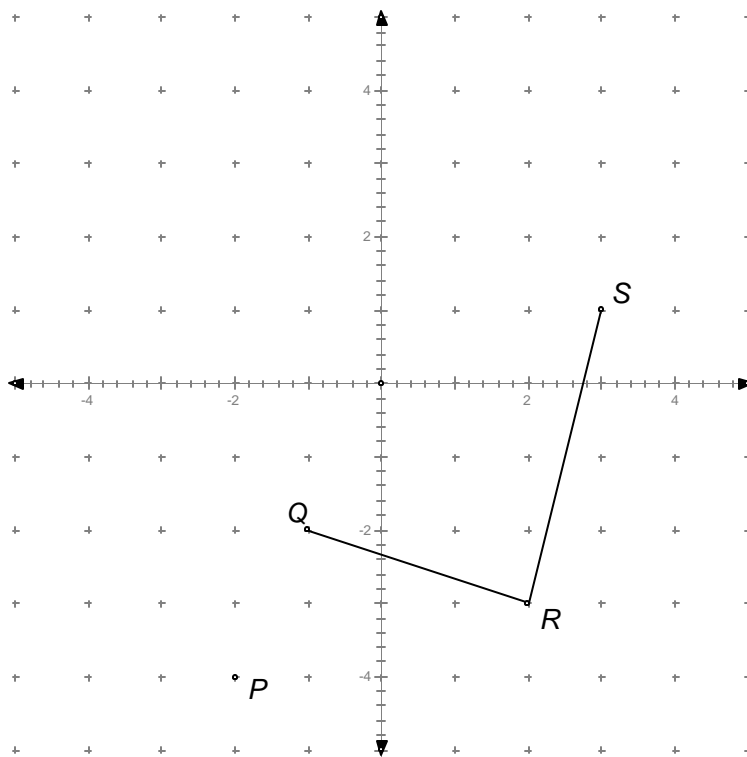


Figure 1:

What is the domain of  $-g(1 - x) + 1$ ?

What is the range of  $-g(1 - x) + 1$ ?

Find  $(g \circ g)(2)$

Sketch the graph of  $y = -g(1 - x) + 1$  on Figure 1. Label the points where P, Q, R and S end up at as  $P_1$ ,  $Q_1$ ,  $R_1$  and  $S_1$

What is the domain of the function  $h(t) = \frac{1}{\sqrt{2 + t - t^2}}$

Figure 2 below shows the graph the function,  $F(x)$ . Refer to this figure for the next three questions

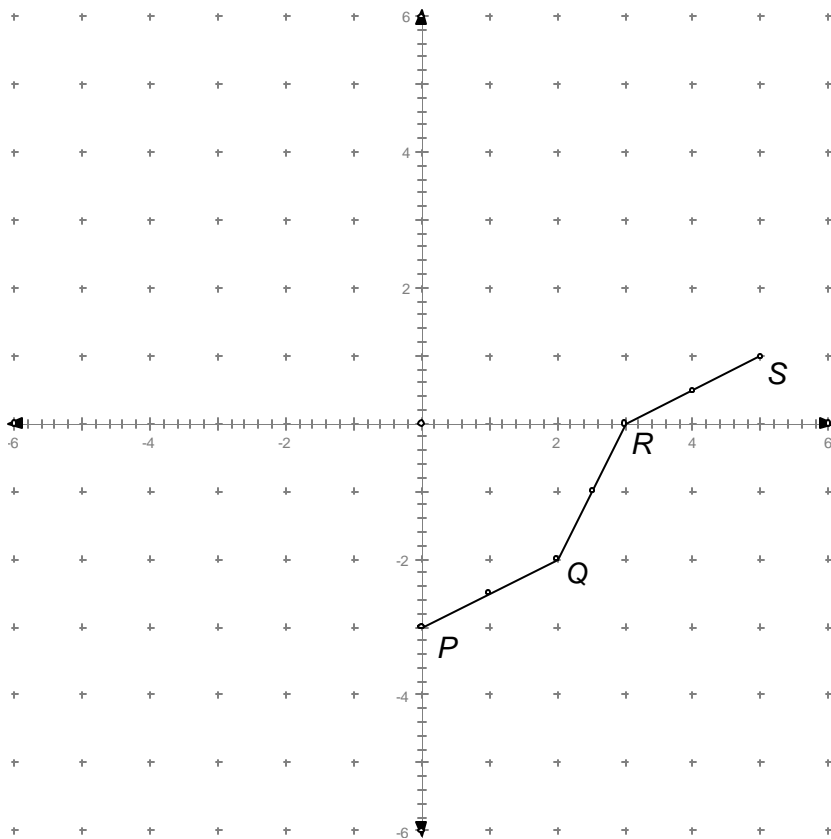


Figure 2:

Sketch the graph of  $-F^{-1}(x + 1) + 1$  on figure 2. Label the new locations of the points P, Q, R and S as  $P_1$ ,  $Q_1$ ,  $R_1$  and  $S_1$  along with their new coordinates.

Find the range and domain of  $F^{-1}(x)$

Find the range and domain of  $-F^{-1}(x + 1) + 1$

If  $A$  varies directly as the square root of  $D$  and inversely as the square of  $C$ . Write down the law of variation for this statement. What happens to  $A$  if  $C$  is halved and  $D$  is quadrupled?

Find  $t$  if  $2f\left(\frac{t+2}{1-t}\right) - 8 = 4$  and  $f^{-1}(6) = -1$

Find,  $f^{-1}(x)$  for the the function  $f(t) = \frac{2x+1}{2+x}$ . If  $f^{-1}$  does not exist, explain. If  $f^{-1}$  exists, what is its domain and range?

Let  $g(x) = 2x^2 - 3x + 1$ . Compute the difference quotient  $\frac{g(x+h) - g(x)}{h}$

Sketch the graph of  $f(x) = \begin{cases} x + 1, & -3 \leq x < -2 \\ -1, & -2 \leq x < -1 \\ 0, & x = 0 \\ 1, & 1 < x \leq 2 \\ -x + 1, & 2 < x \leq 3 \end{cases}$

Figure 1 below is the graph of a function  $g(x)$ , the points P, Q, R and S lie on the this graph. Refer to this figure for the next four questions.

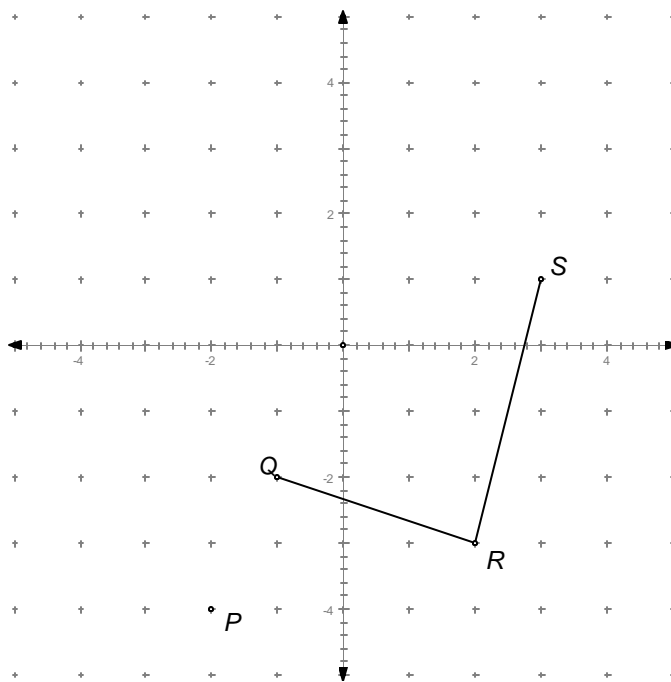


Figure 1:

What is the domain of  $g(x)$ ?

What is the range of  $g(x)$ ?

Evaluate  $(g \circ g)(2)$  and  $(g \circ g)(-1)$

Sketch the graph of  $y = -g(x + 1) + 1$  on Figure 1. Label the points where P, Q, R and S end up at as  $P_1$ ,  $Q_1$ ,  $R_1$  and  $S_1$  along with their new coordinates

What is the domain of the function  $h(t) = \frac{1}{\sqrt{2-t-t^2}}$ . Give exact answers (not graphical) and write your answer in interval notation.

Figure 2 below shows the graph of the function,  $F(x)$ . Refer to this figure for the next two questions.

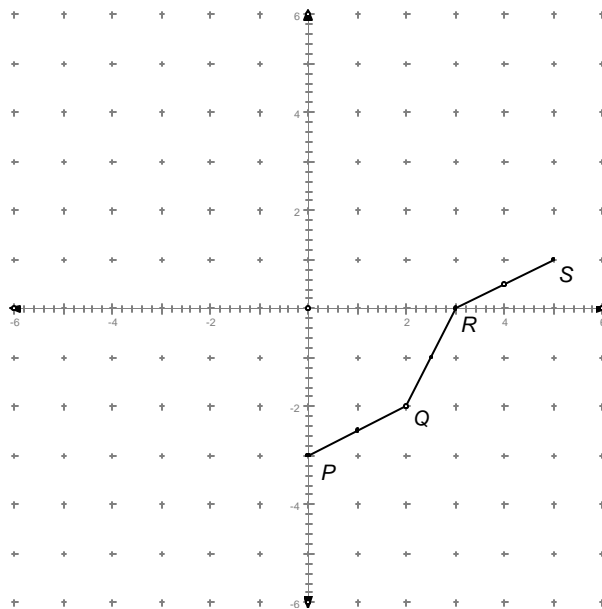


Figure 2:

Sketch the graph of  $F^{-1}(x)$  on Figure 2. Label the new locations of the points P, Q, R and S as  $P_1$ ,  $Q_1$ ,  $R_1$  and  $S_1$  along with their new coordinates. What is the domain and range of  $F^{-1}(x)$  ?

What is the domain and range of  $F^{-1}(x + 1) - 3$

Find  $t$  if  $3f\left(\frac{t+2}{1-t}\right) - 8 = 4$  and  $f^{-1}(4) = -2$

If  $f(x) = 3x^2 + 2x - 1$  and  $g(x) = 2x - 1$ , find and simplify  $(f \circ g)(x)$ .

Let  $g(x) = 2x^2 + 3x - 1$ . Compute the difference quotient  $\frac{g(x+h) - g(x)}{h}$

Find,  $f^{-1}(x)$  for the the function  $f(x) = \frac{2x-1}{2-x}$ . If  $f^{-1}$  does not exist, explain. If  $f^{-1}$  exists, what is its domain and range?

Given that  $(f \circ g)(x) = 20x - 6$  and  $f(x) = 2x + 1$ , find  $g(x)$ .

Let  $f(x) = x + 3$  and  $g(x) = x^2 + x$ .  
Find and simplify  $(g \circ f)(x)$ .

Simplify  $\frac{g(x+h) - g(x)}{h}$  when  $g(x) = x^2 + x$ .

Let  $f(x) = \sqrt{x} + 2$ .

i) Find the domain and range of  $f(x)$ .

ii) Find  $f^{-1}(x)$ .

Let  $f = \{(-2, 4), (-3, 5), (1, -3)\}$  and  $g = \{(-2, 4), (4, -3), (-3, 2)\}$ .

i) Find  $fg$ .

ii) Find  $g \circ f$ .

Let  $f(x) = \frac{1}{\sqrt{x}}$  and  $g(x) = \frac{1}{x^2} + 3$ .

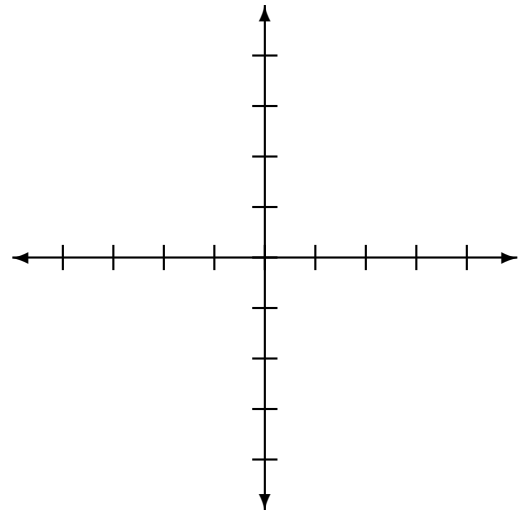
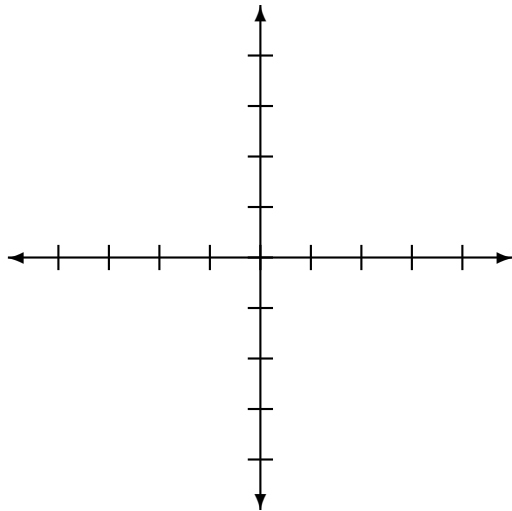
Find and simplify  $(g \circ f)(x)$ .

Make sure you specify the domain of the function.

If  $(f \circ g)(x) = 6x + 7$  and  $f(x) = 3x + 4$  then find  $g(x)$ .

Given that the domains of  $f(x)$  and  $f^{-1}(x)$  are both all real numbers and that  $f(4) = 7$ , solve the equation  $2f^{-1}(2t + 4) - 6 = 2$  for  $t$ .

The graph of  $h(x)$  is the line segment from  $(1, 2)$  to  $(-2, 1)$ .  
Graph  $h^{-1}(x)$  and  $h(1 - x) - 2$ . (Be sure to label endpoints.)



Given that the point  $(-2, 6)$  is a point on the graph of  $y = f^{-1}(x)$ , find a point on the graph of  $y = -3f(2x) + 4$ .

Find the domain of  $f(x) = \frac{x + 2}{\sqrt{x^2 + 2x - 15}}$ .

The velocity of an earth satellite in a circular orbit varies inversely as the square root of the radius of the orbit. What happens to the velocity if the radius is cut in half?

Simplify  $\frac{f(x+h) - f(x)}{h}$  when  $f(x) = (x+1)^2$ .

Let  $f(x) = \frac{2x-1}{x+3}$

i) Find the domain and range of  $f(x)$ .

ii) Find  $f^{-1}(x)$ .

Let  $f = \{(-2, 4), (-3, 5), (1, -2)\}$  and  $g = \{(-2, 4), (4, 1), (-3, 2)\}$ .

i) Find  $f + g$ .

ii) Find  $g \circ f$ .

Let  $f(x) = \sqrt{x}$  and  $g(x) = \sqrt{x^2 + 3}$ .

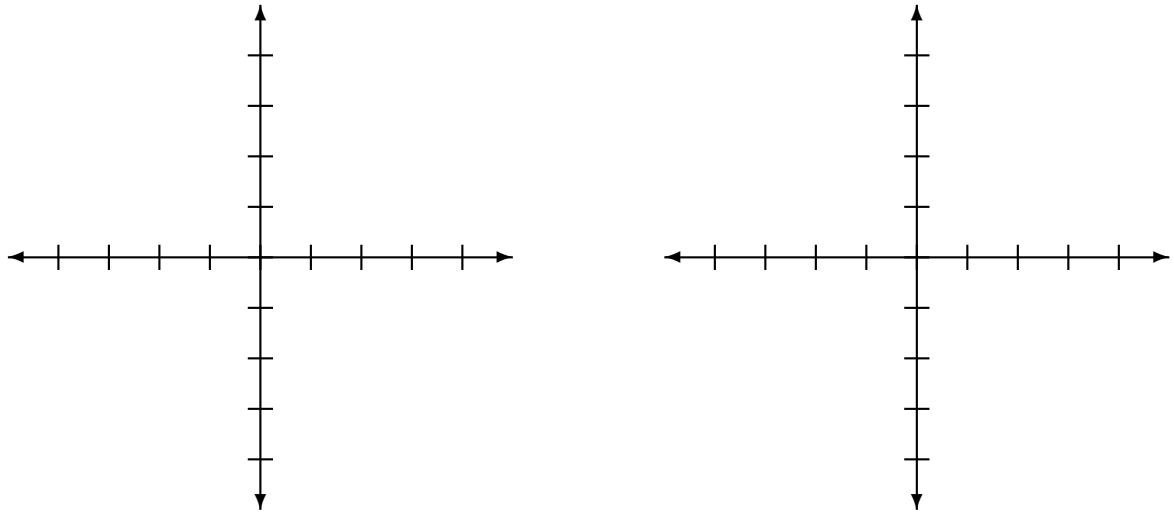
Find and simplify  $(f \circ g)(x)$ .

Make sure you specify the domain of the function.

Find the domain of the following function:  $f(x) = \sqrt{x^2 - x - 6}$

Given that the domains of  $f(x)$  and  $f^{-1}(x)$  are both all real numbers and that  $f^{-1}(1) = 7$ , solve the equation  $3f(2t+8) - 1 = 2$  for  $t$ .

The graph of  $h(x)$  is the line segment from  $(2, 1)$  to  $(-1, 3)$ .  
 Graph  $h^{-1}(x)$  and  $h(1-x)$ . (Be sure to label endpoints.)



Given that the point  $(-2, 6)$  is a point on the graph of  $y = f^{-1}(x)$ , find a point on the graph of  $y = -3f(2x) + 4$ .

Sketch the graph of the following function on the axis provided and state the domain.

$$f(x) = \begin{cases} x^2 - 2 & \text{if } x < -2 \\ \sqrt{x-1} & \text{if } x \geq 1 \end{cases}$$

